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RESEARCH DEPARTMENT

The choice of interpolation apertures for line-store standards converters

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SUMMARY

This report is concerned with the manner in which the output signal from a line-store standards converter is influenced by the type of interpolation aperture which is used in deriving this output signal. The optimum shape of the interpolation aperture is deduced theoretically for apertures of various widths and an experiment is described which was carried out to examine the agreement between these theoretical arguments and practical results.

1. INTRODUCTION

In order that a display of the output signal from a line-store standards converter should be as similar as possible to that obtained by direct scanning of the original picture, the information on each output line must be derived by interpolating between the information presented by at least two successive input lines.

The relation between the relative positions of the input and output scanning lines and the proportion of the output signal contributed by each input line is known as the interpolation law.[1] Variation of this interpolation law influences the display of a converted signal in two ways, namely it affects,

(*a*) the spectrum of wanted vertical detail. (Vertical detail is defined as the variations in brightness along a vertical line down a picture.)

(*b*) the amount of spurious vertical detail which is generated in the converter.

The purpose of this report is to determine the interpolation law which should be used in order to obtain the optimum results with respect to the two factors given above.

Only interpolation between input lines which are in the same field has been considered in this report. While interpolation between successive picture lines would give more acceptable results, it is far more difficult to perform because video-signal delays of the order of a field-period instead of a line-period are required. The results given in this report can easily be applied, however, to interpolation between successive picture lines.

2. TERMINOLOGY

It is convenient, at this point, to define some of the terms, abbreviations and symbols used in this report.

The terms "horizontal direction" and "vertical direction" refer to the directions of line and field scanning respectively.

Where no further indication has been given, the terms "input" and "output" refer to the input and output of a standards converter.

The following abbreviations and symbols are used throughout the report:

p.h. Picture height. (This is used as the unit of length in the vertical direction of a displayed picture. The distances between points on a vertical line have been given as a fraction of the total picture height.)

d. This symbol is used as an independent variable representing distance measured down a vertical line on a picture.

c/p.h. Cycles per picture height. (The spatial frequency of a pattern which is periodic in the vertical direction has been stated in cycles per picture height.)

n_i The number of lines in one field of the television waveform applied to the input of a standards converter (e.g. $n_i = 202\frac{1}{2}$ if this standard is the British 405-line system).

n_o The number of lines in one field of the television waveform at the output of a standards converter.

It follows, from the above definitions, that if field blanking is neglected, the distance between two successive scanning lines in one field of the input standard is $(1/n_i)$ p.h.; the corresponding distance for the output standard is $(1/n_o)$ p.h.

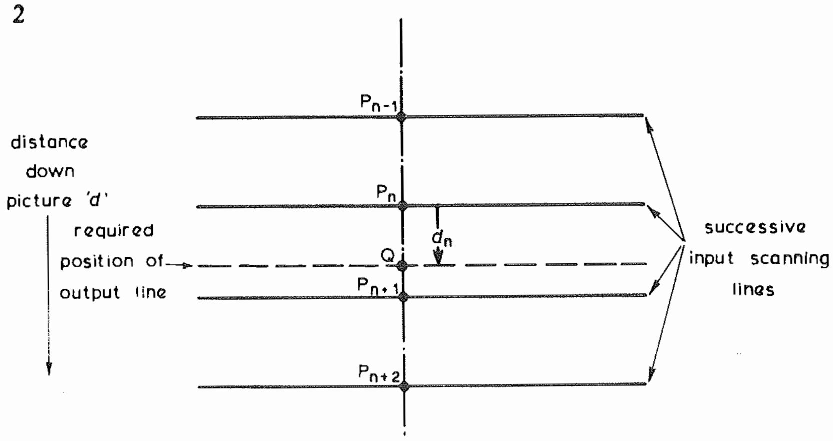

Fig. 1

Relative positions of input and output lines

3. BASIC THEORY OF INTERPOLATION

The principles of interpolation in television standards conversion have already been described elsewhere,[1,2,3] but for convenience the principles relevant to this report are repeated below.

In general, each scanning line required by the output standard of a converter will lie in between the positions occupied by input standard scanning lines. In all existing line-store converters, the magnitude of the output signal at the instant the output standard scans a given point Q (see Fig. 1) is derived by interpolating between the input signals corresponding to points lying vertically above and below point Q, e.g. points P_n, P_{n+1} etc. Thus the video signal for each output line is derived by adding together prescribed proportions of video signals from two or more input lines.

The proportion $S[d]$ of the video signal taken from any one input line is a function of its distance 'd' from the output line to which it is contributing. By analogy with effects in optical and electron-optical systems, the manner in which $S[d]$ varies with 'd' is regarded as determined by an "interpolation aperture". One possible interpolation aperture is "cosine-squared" aperture having the shape shown in Fig. 2.

If the output signal is derived from only the two input lines which lie on either side of each output line, (i.e. if $S[d] = 0$ for $|d| > 1/n_i$), then the magnitude e_Q of the output signal at point Q in Fig. 1 is given by:

$$e_Q = S[d_n]\ e_n + S[d_n - 1/n_i]\ e_{n+1} \tag{1}$$

where e_n and e_{n+1} are the magnitudes of the input signal at points P_n and P_{n+1} and d_n is the distance between from P_n to Q.

One fairly obvious requirement of such an interpolation aperture is that if $e_n = e_{n+1}$ then e_Q should equal both e_n and e_{n+1} for all positions of Q between P_n and P_{n+1}. Failure to meet this requirement would cause a pattern of horizontal stripes in the converted picture.

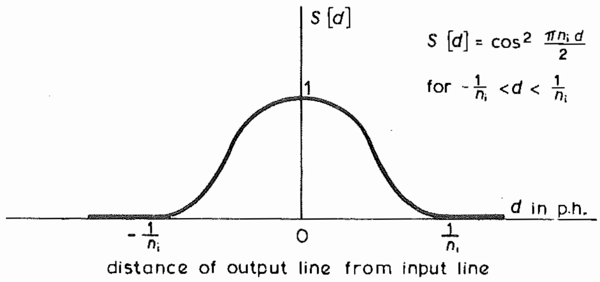

Fig. 2 - "Cosine-squared" interpolation aperture

Substituting this condition in equation (1) gives that:

$$S[d_n] + S[d_n - 1/n_i] = 1 \tag{2}$$

This equation defines the shape of the interpolation aperture for negative values of 'd' when its shape is given for positive values of 'd' or vice versa.

The manner in which an interpolated output signal is derived will now be considered by reference to Fig. 3.

Suppose the brightness along a narrow vertical section of the scene scanned by the input standard varies in the manner shown in Fig. 3(*a*). Also suppose this vertical section is crossed by input scanning lines at points P_1, P_2, P_3, etc. and by output lines at points Q_1, Q_2, Q_3, etc. The relative magnitudes e_1, e_2, e_3, etc. of the input signal corresponding to points P_1, P_2, P_3, etc. will be a function of the brightness of the scene at these points and will therefore be as shown in Fig. 3(*b*).

An interpolated envelope of these input line-samples, i.e. 'e_Q' as a continuous function of 'd', may be obtained by first replacing each of the samples by a curve of the same shape as the interpolation aperture (see Fig. 3(*c*)). The interpolated envelope of the input line-samples is given by the sum of these separate curves and is indicated by the full line in Fig. 3(*c*). The relative magnitudes of the output signal e'_1, e'_2, e'_3 at points Q_1, Q_2, Q_3, etc. are given by the magnitudes of the interpolated envelope at these positions and are therefore as indicated in Fig. 3(*d*).

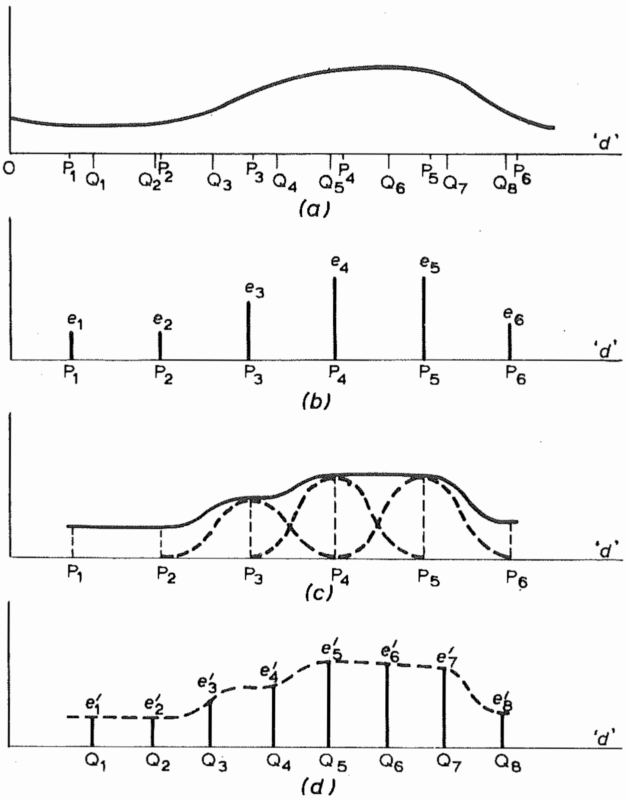

Fig. 3 - Derivation of the magnitude of the output signal from a converter at points along a narrow vertical strip of picture

(*a*) Brightness of original scene
(*b*) Magnitude of input line samples
(*c*) Magnitude of interpolated envelope of input line samples
(*d*) Magnitude of output line samples

Since the output signal obtained in the manner described above is obtained by sampling an interpolated envelope of the input signal samples instead of by sampling the brightness of the original scene, it follows that if the shape of the curve representing the interpolated envelope of input line samples is identical to the shape of the curve representing the brightness of the original scene, then the output signal obtained will be identical to that obtained by direct scanning of the original scene at the output standard. In general, however, it is not possible to obtain an interpolation aperture which results in these two curves being identical and therefore the display of the converted signal is normally impaired to a certain extent by the conversion process. In order to analyse the effects of imperfect interpolation, it is more convenient to compare the frequency spectra of the waveforms shown in Fig. 4 rather than compare the variations of magnitude with distance which they represent. These frequency spectra will therefore now be considered.

4. VERTICAL SPECTRA

4.1. Derivation of the Vertical Spectrum of a Display of a Converted Signal

If the frequency spectrum of the brightness variations along a vertical section of the original scene is known, then the frequency spectrum of the brightness variations down the corresponding vertical section on a display of the converted signal may be derived as follows:

Suppose the vertical spectrum of the original scene illustrated in Fig. 3(*a*) is as shown in Fig. 4(*a*), where a component at frequency p c/p.h. has been marked for reference purposes.

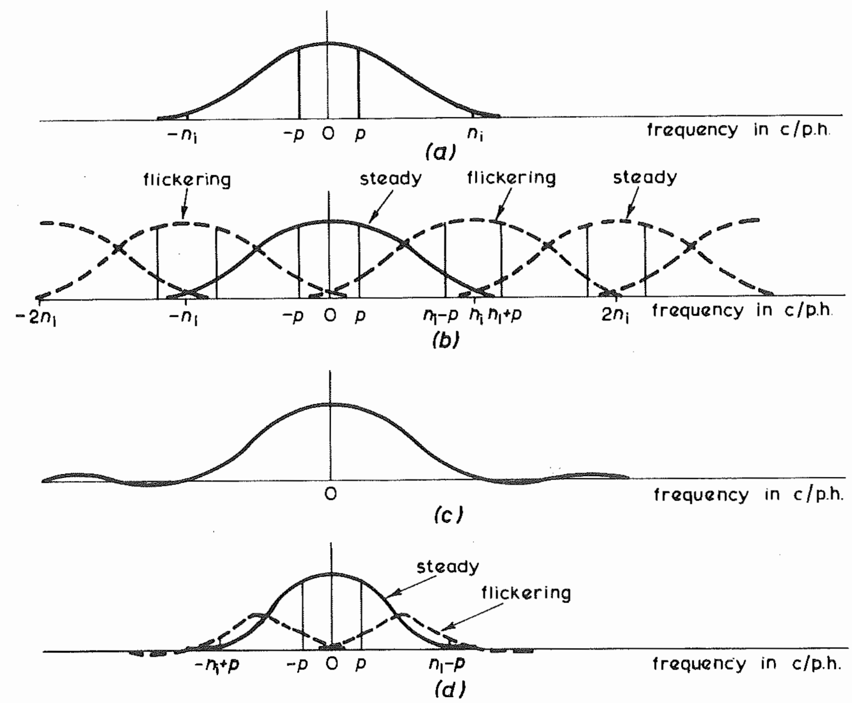

Fig. 4 - Derivation of vertical spectrum of interpolated envelope of input samples

(*a*) Spectrum of original scene (*b*) Spectrum of input line samples
(*c*) Spectrum of 'cosine-squared' interpolation (*d*) Spectrum of interpolated envelope of input line samples

From well known theory, it can be shown that sampling the brightness of the original scene at intervals of $1/n_i$ p.h. has the effect of adding to the spectrum of this scene displaced versions of it spaced in frequency by n_i c/p.h. The vertical spectrum of the input line samples shown in Fig. 3(*b*) is therefore as shown in Fig. 4(*b*). The full line in Fig. 4(*b*) represents frequency components appearing in the spectrum of the original scene, while the broken lines represent spurious components produced by the scanning process.

The annotation "steady" and "flickering" on Fig. 4 will be referred to in Section 4.2.

In order to determine the spectrum of the interpolated envelope of input line samples shown in Fig. 3(*c*), it is first necessary to obtain the spectrum of the interpolation aperture used to construct this envelope. The spectrum of the "cosine-squared" interpolation aperture used to obtain Fig. 3(*c*) is shown in Fig. 4(*c*).

Considering, now, the interpolated envelope of input line samples shown in Fig. 3(*c*), this waveform was obtained by replacing each of the infinitely narrow pulses in the waveform shown in Fig. 3(*b*) by a broad pulse of the same shape as the interpolation aperture. Since an infinitely narrow pulse has a flat frequency spectrum, and a broad pulse has the spectrum shown in Fig. 4(*c*), the process of obtaining the interpolated envelope of input line samples is equivalent to passing the waveform representing the input line samples through a filter with a response/frequency characteristic of the same shape as the spectrum of the interpolation envelope. In other words, the amplitude of the spectrum of the interpolated envelope at a particular frequency is proportional to the product of the amplitudes of the spectra shown in Fig. 4(*b*) and (*c*) at that frequency; the spectrum thus obtained is shown in Fig. 4(*d*).

The spectrum of the output line samples shown in Fig. 3(*d*) may now be derived by means of the same theory used to derive the spectrum of the input line samples. From this theory, it can be shown that sampling the interpolated envelope of input line samples at intervals of $1/n_o$ p.h. has the effect of adding to the spectrum of this interpolated envelope displaced versions of it spaced in frequency by n_o c/p.h. The spectrum of the output line samples shown in Fig. 3(*d*) is therefore given by the sum of the spectra shown in Figs. 5(*b*), (*c*) and (*d*) plus higher frequency components not shown.

The output line samples represent the brightness variations down a vertical section of a display of the converted signal (assuming an infinitely narrow scanning beam) and therefore Figs. 5(*b*), (*c*) and (*d*) represent the vertical spectrum of such a display.

For purposes of comparison, the vertical spectrum of a display obtained by direct scanning of the original scene at the output standard is shown in Fig. 5(*e*).

4.2. Effect of Interlacing on Vertical Spectra

Up to this point the discussion has applied only to a single field and the effect of interlacing has been ignored. If the input and output standards are interlaced systems, then the sampling pulses used to derive the input and output line samples change in position during alternate fields as shown in Fig. 6(*a*). The spectra of the two trains of sampling pulses shown in Fig. 6(*a*) are shown in Fig. 6(*b*). It can be seen from Fig. 6(*b*) that interlacing causes the components at odd integral multiples of n c/p.h. to change in phase by 180° during alternate field periods. When the input standard is interlaced, the alternation in phase of these components in the spectrum of the input sampling pulses causes a corresponding alternation in phase of the parts of the spectra of the input line samples which are indicated by curves centred on odd integral multiples of n_i c/p.h. (see Fig. 4(*b*)). Similarly, the parts of the spectrum of the output line samples indicated by curves centred on odd integral multiples of n_o in Fig. 5 also alternate in phase when the output standard is interlaced. If the field frequency is 50 Hz, then this phase alternation occurs at a repetition rate of 25 Hz, and components which alter in phase in this way appear to flicker at 25 Hz when displayed on a monitor.

The components in the spectra of a display of a converted picture which alternate in phase when converting between interlaced systems have been marked "Flickering" in Fig. 5 and the remaining components have been marked "Steady".

4.3. Comparison of Vertical Spectra of Converted Picture and of Original Scene

The impairment of the display of the converted signal resulting from the scanning and conversion processes will now be considered.

First, the wanted information in the vertical spectrum of the display of the converted signal, given by the full line in Fig. 5(*b*), is attenuated at high frequencies according to the spectrum of the interpolation function shown in Fig. 4(*c*).

Secondly, this spectrum of the converted signal contains spurious components introduced by the two scanning processes. The lower frequency spurious components may conveniently be divided into the following groups:

(*a*) unwanted components produced by the input scanning process which are displaced in frequency by n_i c/p.h. These components are indicated by the broken lines in Fig. 5(*b*) and alternate in phase when the input scanning lines are interlaced.

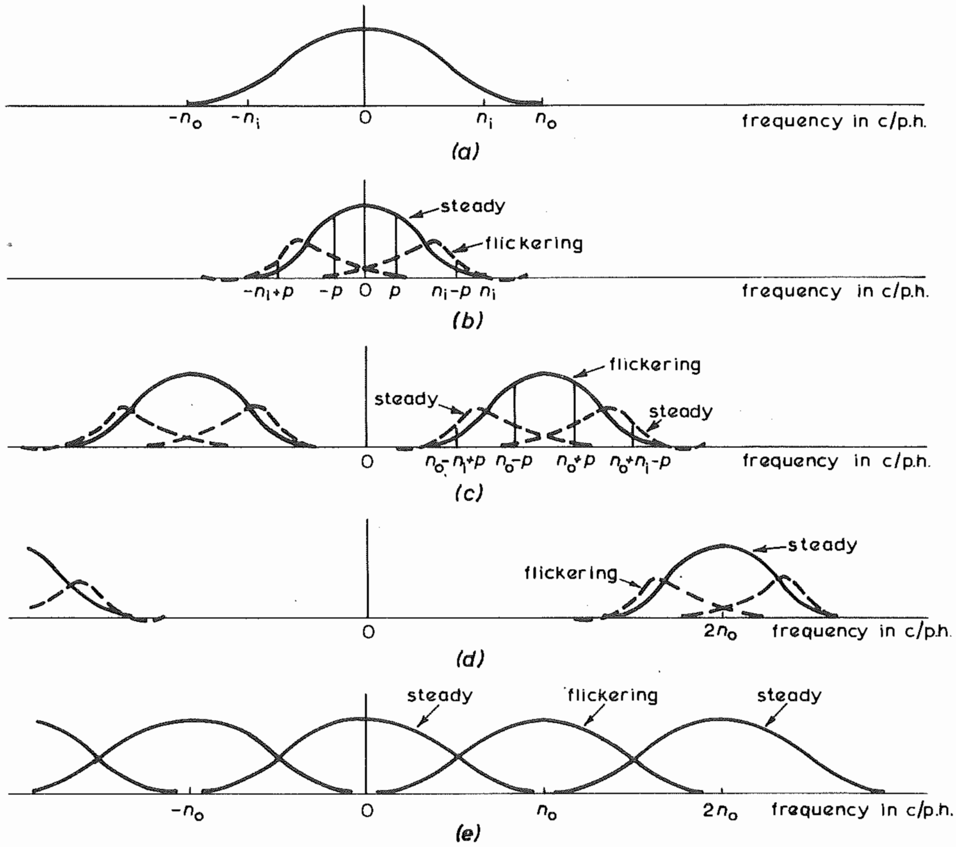

Fig. 5 - Vertical spectra of displays of converted signal and signal obtained by direct scanning at output standard

(a) Original scene (b), (c) and (d) Display of converted signal

(e) Display obtained by direct scanning at output standard

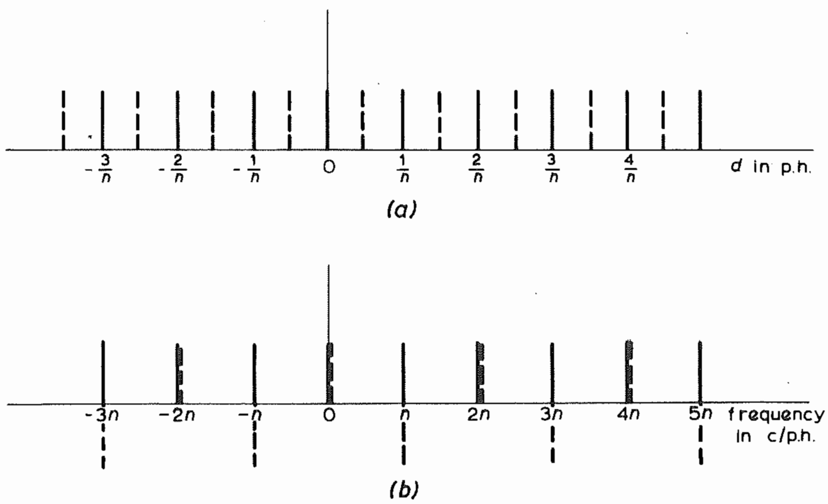

Fig. 6 - Effect of interlace on position and spectrum of sampling pulses

(a) Alternative positions of sampling pulses

——— Position during even fields - - - - - Position during odd fields

(b) Alternative spectra of sampling pulses

——— Spectrum during even fields - - - - - Spectrum during odd fields

(*b*) unwanted components produced by the output standard scanning process which are displaced in frequency by n_o c/p.h.

These components are indicated by the full line in Fig. 5(*c*) and alternate in phase when the output scanning lines are interlaced. (Such components are also present in a display obtained by direct scanning at the output standard.)

(*c*) unwanted components produced by a combination of the input and output scanning process which are displaced in frequency by ($n_i \pm n_o$) c/p.h.

These components are indicated by the broken lines in Fig. 5(*c*). These components do not alternate in phase when both input and output scanning lines are interlaced because the alternation in phase due to the input scanning process is cancelled by the alternation due to the output scanning process.

When both input and output standards are interlaced systems, the unwanted components of types (*a*) and (*b*) flicker at 25 Hz and are probably less objectionable than the unwanted components of type (*c*) which are steady in a still picture but flicker slowly with slow vertical movement.

The purpose of the following sections in this report is to determine the shape and spectrum of the interpolation aperture which corresponds to minimum impairment of the display of the converted signal.

5. "IDEAL" INTERPOLATION APERTURE

It was shown in Section 3 that if the waveform (see Fig. 3(*c*)) representing the interpolated envelope of input line samples is identical to that of the original scene (see Fig. 3(*a*)), then the display of the converted signal will be identical to the display obtained by direct scanning at the output standard. It therefore follows that a most important factor to be considered in the determination of the best possible or "ideal" interpolation aperture is that these two waveforms and therefore their spectra should be as similar as possible. These two spectra can only be made identical if,

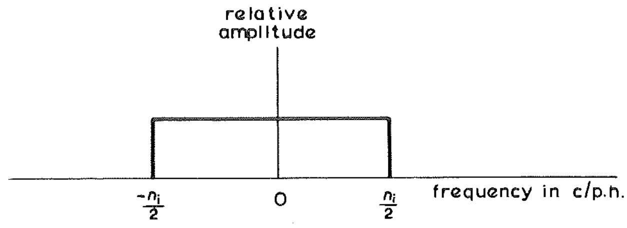

Fig. 7 - Spectrum of "Ideal" interpolation aperture

(*a*) the spectrum of the original scene contains no components above $n_i/2$ c/p.h.;

and

(*b*) the amplitude of the spectrum of the interpolation aperture is flat between $\pm\, n_i/2$ c/p.h. and is zero outside these limits, as shown in Fig. 7.

Condition (*a*) cannot be controlled by any process in a standards convector and therefore it is not always possible to select an interpolation aperture which will make the waveform in Fig. 3(*c*) identical to that shown in Fig. 3(*a*). It is still of interest however to examine the results obtained with an interpolation aperture having the spectrum shown in Fig. 7. The shape of the interpolation aperture corresponding to this spectrum is shown in Fig. 8 and is represented by the equation:

$$S(d) = \frac{\sin \pi n_i d}{\pi n_i d} \tag{3}$$

By reference to Fig. 4, it can be seen that this interpolation aperture will remove wanted detail having vertical spatial frequencies above $n_i/2$ c/p.h. but will not remove the corresponding unwanted components which have frequencies lower than $n_i/2$ c/p.h. Also, when the amplitude of spectrum shown in Fig. 4(*a*) falls with increasing frequency as shown, then raising the cut-off frequency of the interpolation aperture above $n_i/2$ c/p.h. introduces more unwanted than wanted components into the spectrum shown in Fig. 4(*d*) while lowering the cut-off frequency removes more wanted than unwanted information.

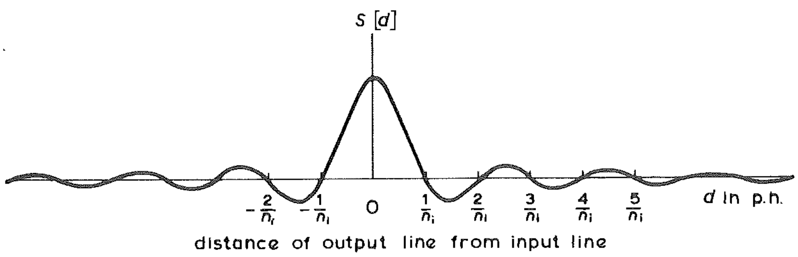

Fig. 8 - Shape of "Ideal" interpolation aperture

It would therefore appear that the interpolation aperture shown in Fig. 8 is the best possible or "ideal" aperture even when the vertical spatial spectrum of the original scene has components above $n_i/2$ c/p.h. There is, however, a further factor which affects the selection of the "ideal" interpolation aperture. This arises from the fact that the output standard is not capable of correctly portraying vertical detail with a higher frequency than n_o c/p.h. (assuming the output standard is an interlaced system). Therefore if $n_o < n_i/2$, the display of the converted signal will be improved if the cut-off frequency of the spectrum of the interpolation aperture is lowered from $n_i/2$ c/p.h. to n_o c/p.h. (Because of imperfections in interlacing, better results will probably be obtained if this cut-off frequency is somewhat lower than n_o c/p.h.)

Since the ideal aperture shown in Fig. 8 has infinite width, its use as an interpolating aperture demands that the signal required during an output line should be derived from the signals carried by an infinite number of input lines. It is, therefore, impossible to obtain this aperture in practice, but a knowledge of the ideal spectrum is a useful guide when considering the compromises necessitated by apertures of limited width.

6. OPTIMUM INTERPOLATION APERTURE OBTAINABLE IN PRACTICE

6.1. General

In this Section, interpolation apertures of limited width are considered.

It has already been made clear that the treatment of vertical detail by interpolation is best considered in terms of the spectrum of the interpolation aperture, that is, its Fourier transform. We now have the problem of restricting considerations to those spectra which are derived from apertures wholly confined within some finite width. The procedure adopted is to regard the aperture as the product of a periodic function and a single square pulse that selects only one cycle of this function. This procedure is clarified below by application to a particular case.

6.2. Symmetrical Apertures Limited in Width to Twice the Distance Between Successive Input Lines

The aperture shown in Fig. 9(*c*) will be considered as a typical example of a symmetrical interpolation aperture which is limited in width to twice the distance between successive input scanning lines, i.e. to $2/n_i$ p.h. A repetitive waveform, in which each cycle has the same shape as this aperture is shown in Fig. 9(*a*). If $f(d)$ represents the magnitude of this repetitive waveform at any distance 'd' along the horizontal axis of Fig. 9(*a*), then by Fourier analysis, a general expression for $f(d)$ is given by:

$$f(d) = A_o + A_1 \cos\{\pi n_i d\} + \ldots A_r \cos\{\pi m_i d\} \quad \text{etc.} \qquad (4)$$

A_r representing the amplitude of the term with a repetition frequency of $m_i/2$ c/p.h.

The effect on both the shape and the spectrum of the aperture of varying any of the coefficients A_o, A_1, A_r, etc. can be determined relatively easily; these relationships, therefore, provide a useful method of examining the way in which a change in the shape of an aperture will affect its spectrum. The effect which varying A_1, A_2, A_3, etc. has on the spectrum of the aperture will now be considered.

Referring to Fig. 9, it can be seen that a waveform representing the aperture being considered can be obtained by multiplying the repetitive waveform by the single square pulse shown in this Figure. The spectrum of the aperture can, therefore, be obtained from the spectra of the repetitive waveform and the single square pulse, the method being as follows:

It is convenient to represent the spectrum of the repetitive waveform in terms of positive and negative frequencies, as shown in Fig. 10(*a*). Such a spectrum can be obtained by re-writing equation (4) as:

$$f(d) = A_o + \frac{A_1}{2}\cos(\pi n_i d) + \frac{A_1}{2}\cos(-\pi n_i d) \text{ etc.} \qquad (5)$$

The spectrum of the single square pulse can be obtained by the Fourier transform method and is shown in Fig. 10(*b*) in which the relative amplitude $\phi(f)$ is related to the frequency 'f' (measured in c/p.h.) by the expression:

$$\phi(f) \;\alpha\; \frac{\sin(2\pi f/n_i)}{2\pi f/n_i} \qquad (6)$$

Since the waveform representing the aperture is given by the product of the repetitive waveform and the waveform representing the single square pulse, the spectrum of the aperture is given by convolving the spectra of these two waveforms i.e. it is given by the sum of a series of displaced versions of the of the spectrum of the single square pulse, each version being centred upon and proportional in amplitude to the components in the spectrum of the repetitive waveform (see Fig. 10(*c*) and 10(*d*)).

The spectrum of the interpolation aperture shown in Fig. 10(*d*) can alternatively be represented in terms of positive frequencies only, as shown in Fig. 10(*e*).

It is important to note that the amplitude of the of the spectrum of the single square pulse is zero at all frequencies which are integral multiples of $n_i/2$ c/p.h. except zero. As a result, the relative amplitude of the spectrum of the aperture at a frequency $rn_i/2$ c/p.h., where r is an integer, is directly proportional to A_r, and is independent of the values of A_0, A_1 ... A_{r-1}, A_{r+1}, A_{r+2} etc.

This leads to an interesting general conclusion which may be stated more generally as follows:

If the aperture is restricted to a width equal to or less than w (units of length) then it is possible to specify its spectrum only at a set of discrete ordinates spaced $1/w$ (cycles per unit of length) apart. The value of the spectrum at intermediate points is then determined, and may be obtained by superposing curves of a $(\sin x)/x$ form as described above. This conclusion has other applications e.g. to the synthesis of the radiation patterns of aerials of finite aperture.*

Since it is required that the spectrum of the interpolation aperture should be as close as possible to zero for all frequencies above $n_i/2$ c/p.h., it is reasonable to assume that the values of A_r for $r \geqslant 2$ should all be zero since the spectrum will then have zero amplitude at frequencies $rn_i/2$ c/p.h. for $r \geqslant 2$. In particular, A_r must be zero for all even values of r greater than zero; this condition is essential since the amplitude of the spectrum of the interpolation aperture must be zero at all integral multiples of n_i c/p.h. in order that the interpolation process should completely remove input line frequency and its harmonics. If this condition is not fulfilled, then horizontal striations will appear on the display of the converted signal at all times except when a black scene is being scanned.

* This general theorem was suggested to the author by R.E. Davis.

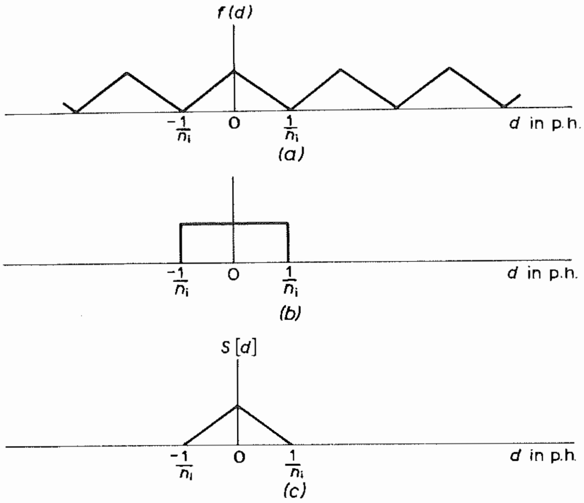

Fig. 9 - Derivation of an interpolation aperture from a repetitive waveform and a single square pulse
(*a*) Repetitive waveform (*b*) Single square pulse
(*c*) Interpolation aperture

If all values of A_r are zero for $r \geqslant 2$ then the corresponding interpolation aperture is represented by:

$$\left.\begin{aligned} S[d] &= A_0 + A_1 \cos \pi n_i d \quad \text{for } -1/n_i < d < 1/n_i \\ S[d] &= 0 \quad \text{for } |d| > 1/n_i \end{aligned}\right\} \tag{7}$$

This aperture is illustrated in Fig. 11, and will be referred to in the remainder of this report as a "cosine plus lift" interpolation aperture.

In order to plot the spectrum of a "cosine plus lift" interpolation aperture for varying values of A_1/A_0 it is convenient first to obtain its spectrum (*a*) when A_0 is finite and $A_1 = 0$ and (*b*) when A_1 is finite and $A_0 = 0$. These two spectra are shown as curves (*a*) and (*b*) in Fig. 12; curve (*a*) corresponds to the curve centred on 0 c/p.h. in Fig. 10(*c*) and curve (*b*) to the sum of the curves centred on $n_i/2$ c/p.h. and $-n_i/2$ c/p.h. The relative amplitudes of curve (*a*) and curve (*b*) have been drawn for equal values of A_1 and A_0. Curve (*c*) in this Figure refers to asymmetrical apertures which will be considered in Section 6.3.

The sum of curves (*a*) and (*b*) gives the spectrum of a "cosine plus lift" interpolation aperture for which $A_1 = A_0$. To obtain the spectrum of a "cosine plus lift" aperture for a different ratio of A_1/A_0, the amplitude of curve (*b*) is altered so that its amplitude at $n_i/2$ c/p.h. is equal to $A_1/2A_0$ times the amplitude curve of (*a*) at 0 c/p.h.; the new curve (*b*) is then added to curve (*a*). The spectra of "cosine plus lift" apertures for which $A_1 = A_0$, $0{\cdot}6A_0$ and $0{\cdot}4A_0$ are plotted in Fig. 13 together with the spectrum of a triangular aperture (as shown in Fig. 9(*c*)).

From Fig. 13 it is clear that the choice of the optimum ratio of A_1/A_0 involves a compromise between the conditions that the spectrum of an aperture should be:

(*a*) as flat as possible up to $n_i/2$ c/p.h. and

(*b*) as close as possible to zero above $n_i/2$ c/p.h.

For example, to satisfy condition (*b*) as closely as possible, the ratio A_1/A_0 should be of the order of 0·5 but condition (*a*) is more closely satisfied when $A_1/A_0 = 1$.

The experiments described in Section 8 of this report were carried out in order to justify imposing condition (*b*) above. It is not possible to determine theoretically the exact value of the optimum ratio of A_1/A_0 since it depends on subjective assessments of the relative impairment of a converted picture caused by the addition of spurious signals and by the attenuation of wanted high frequency vertical detail. The subjective effects observed on a converted picture of varying the ratio A_1/A_0 are considered in Sections 8 and 9 of this report.

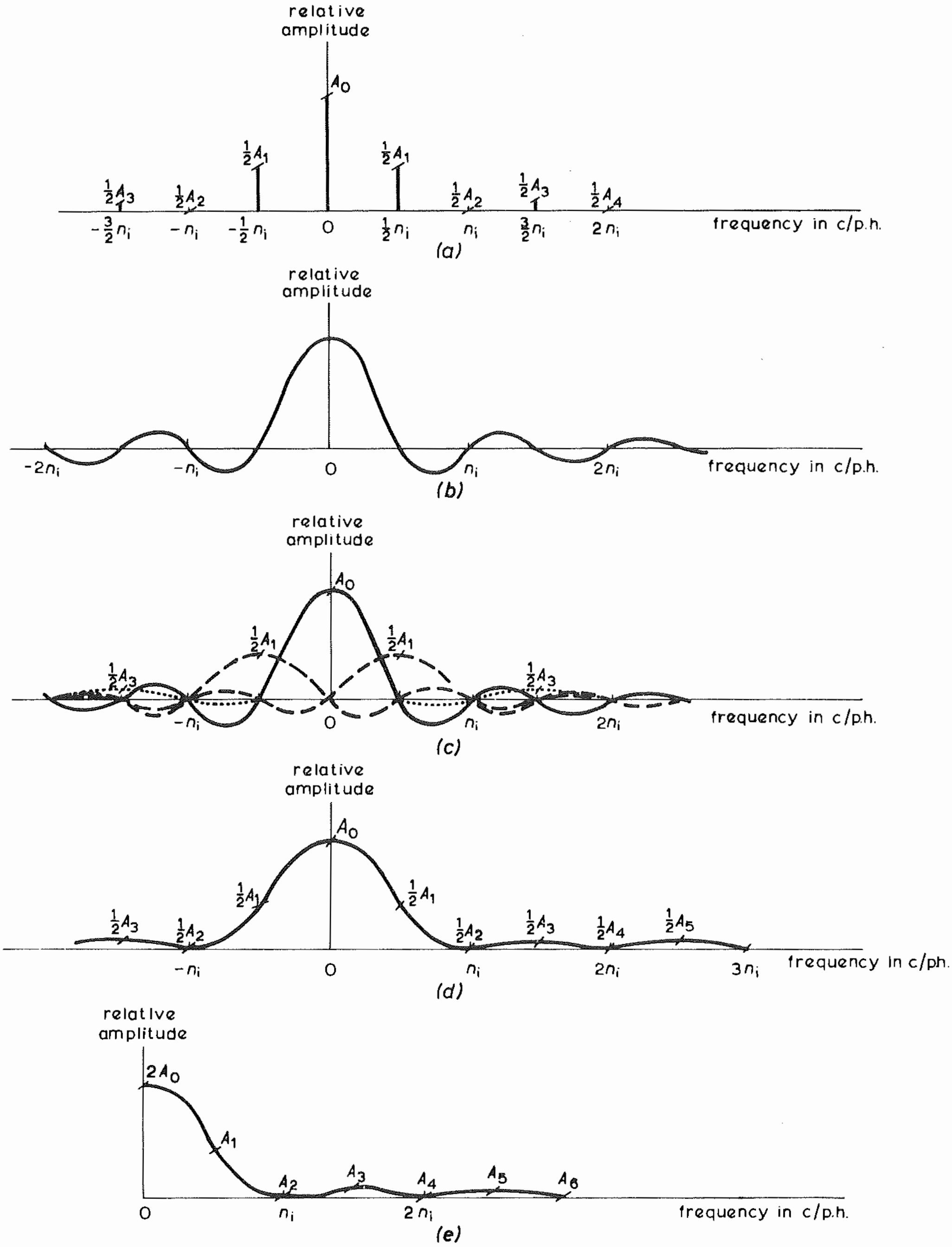

Fig. 10 - Derivation of spectrum of interpolation aperture of width $2/n_i$ p.h.

(a) Spectrum of repetitive waveform (b) Spectrum of single square pulse
(c) Components of spectrum of interpolation aperture (d) Spectrum of interpolation aperture (sum of curves in (c))
(e) Spectrum of interpolation aperture in terms of only positive frequencies

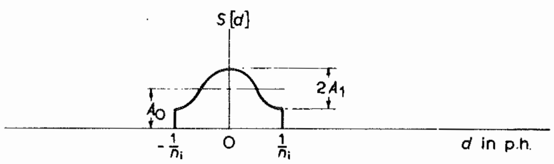

Fig. 11 - "Cosine plus lift" interpolation aperture

By suitably adjusting the ratio of A_1/A_0 the amplitude of the spectrum of a "cosine plus lift" interpolation aperture can be made zero at any frequency between $n_i/2$ c/p.h. and n_i c/p.h. This fact can be used to remove a particular component in the spectrum of the interpolated envelope which lies between these two frequencies.

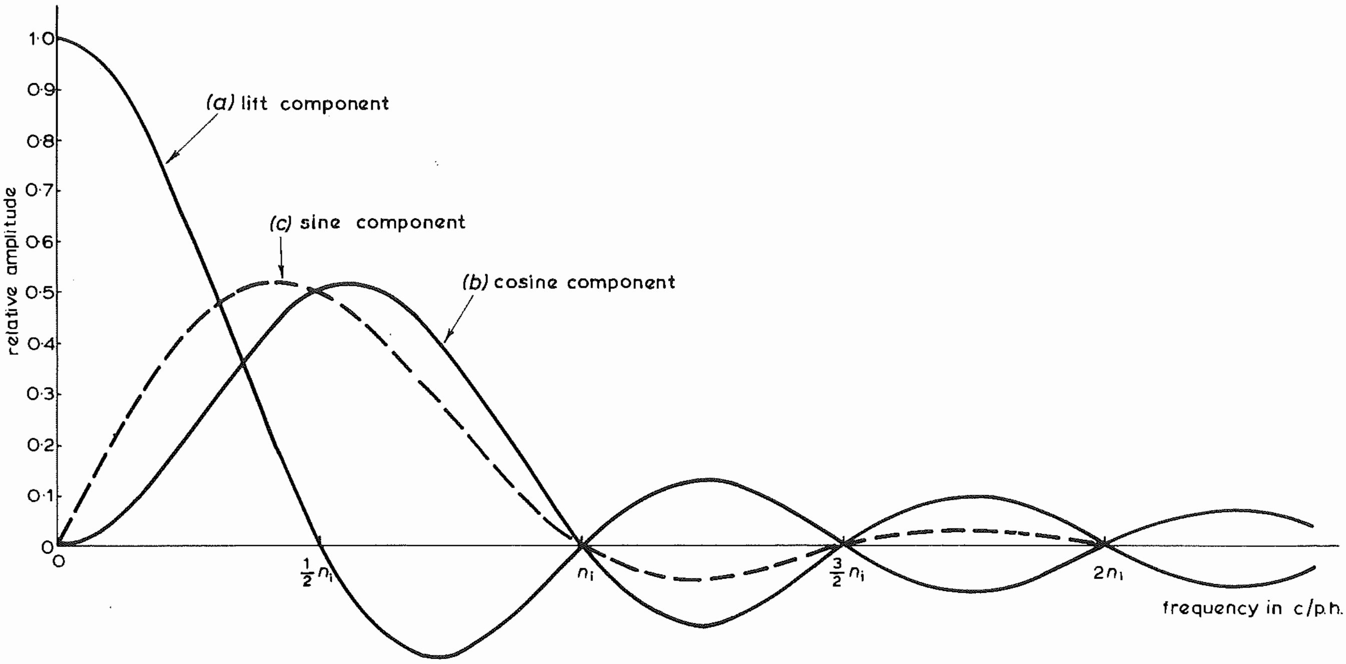

Fig. 12 - Contributions of lift, cosine and sine components of the interpolating aperture to the spectrum of the aperture when $A_0 = A_1 = A_{1q}$

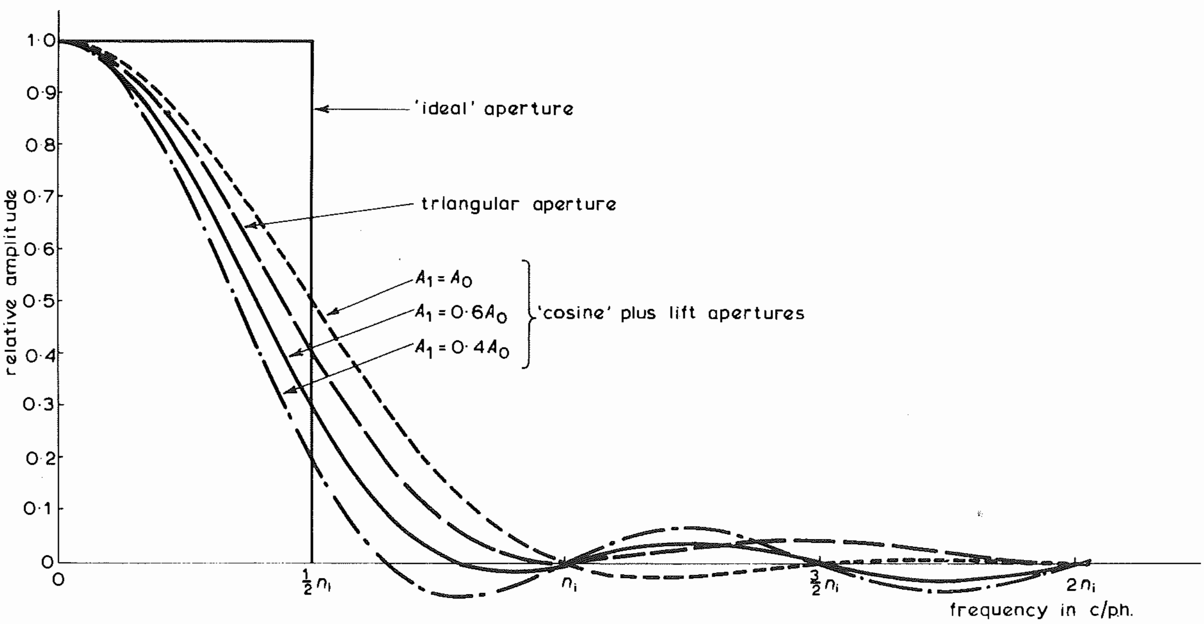

Fig. 13 - Spectra of "cosine plus lift" interpolation apertures for varying ratios of A_1/A_0

Although it has not been proved that a "cosine plus lift" interpolation aperture is quite the best possible aperture for apertures limited in width to $2/n_i$ p.h., it is believed that it leaves little scope for improvement.

6.3. Asymmetrical Apertures Limited in Width to Twice the Distance Between Successive Input Lines

It can be shown from arguments similar to those given above that, if the condition that the interpolation aperture should be symmetrical is removed, then an aperture of width $2/n_i$ p.h.s., which has a spectrum of zero amplitude at n_i, $3n_i/2$, $2n_i$, etc., c/p.h. can be represented by:

$$\left.\begin{array}{l} S[d] = A_0 + A_1 \cos \pi n_i d + A_{1q} \sin \pi n_i d \\ \text{over the range } -\dfrac{1}{n_i} < d < \dfrac{1}{n_i} \\ \text{and by } S[d] = 0 \\ \text{over the range } |d| > \dfrac{1}{n_i} \end{array}\right\} \quad (8)$$

The effect on the spectrum of the interpolation aperture caused by the addition of the term A_{1q} sin $\pi n_i d$ to add quadrature components which vary in amplitude as indicated by curve (*c*) in Fig. 12. The final spectrum of the interpolation aperture at any particular frequency is now given by:

$$|\phi(f)|^2 = (\alpha_0 + \alpha_1)^2 + \alpha_{1q}^2 \quad (9)$$

where α_0, α_1, and α_{1q} are the amplitudes corresponding to curves (*a*), (*b*) and (*c*) respectively at the frequency being considered.

It follows from equation (9) that the addition of of the term A_{1q} sin $\pi n_i d$ in equation (8) causes an increase in the amplitude of the spectrum of the interpolation aperture at all frequencies (except when $\alpha_{1q} = 0$). Since curve (*c*) is approximately symmetrical about $n_i/2$ c/p.h. in the frequency range 0 to n_i c/p.h. this increase will be proportionately greater over the range of $n_i/2$ to n_i c/p.h., where the value of $(\alpha_0 + \alpha_1)$ should be small, than over the range 0 to $n_i/2$ c/p.h. where $(\alpha_0 + \alpha_1)$ should be large. This point is illustrated in Fig. 14 which shows the spectra of two apertures in both of which $A_1 = 0{\cdot}6A_0$ but for one aperture $A_{1q} = 0$ and for the other $A_{1q} = 0{\cdot}8A_0$. Besides having an undesirable effect on the frequency characteristic of the aperture, the asymmetry introduces phase distortion of the wanted frequencies. Therefore for optimum interpolation, an interpolation aperture of width $2/n_i$ p.h. should be symmetrical.

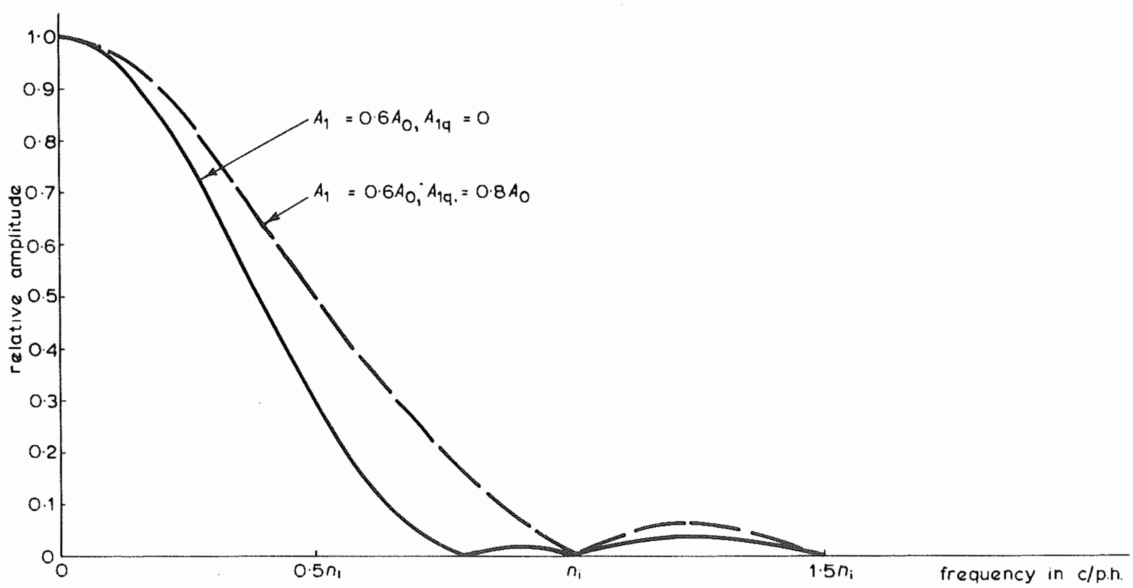

Fig. 14 - Effect of assymmetry in the shape of an aperture on the spectrum of the aperture

6.4. Symmetrical Apertures of Limited Width Greater Than Twice the Distance Between Successive Input Scanning Lines

By similar reasoning to that given for apertures limited in width to $2/n_i$ p.h., it can be shown that, if the interpolation aperture is limited in width to N/n_i p.h. where N is an integer, then the amplitude of the spectrum of the aperture can be specified at all frequencies which are integral multiples of n_i/N c/p.h. For example, if $N = 9$, an aperture of this width can be obtained which has a spectrum passing through the points marked in Fig. 15(*a*), where these points occur at integral multiples of $n_i/9$ c/p.h. The amplitudes of the spectrum at these frequencies have been selected by guesswork as giving a spectrum reasonably close to the "ideal" spectrum shown in Fig. 7.

The spectrum shown in Fig. 15(*a*) corresponds to a repetitive waveform with a period of $9/n_i$ p.h.s. and which is represented by the equation:

$$S(d) = A_0 + A_1 \cos 2\frac{\pi n_i d}{9} + A_2 \cos 4\frac{\pi n_i d}{9}$$

$$+ A_3 \cos 6\frac{\pi n_i d}{9} \text{ etc.} \qquad (10)$$

where the relative values of A_0, A_1, A_2, etc., are as shown in Fig. 15(*a*).

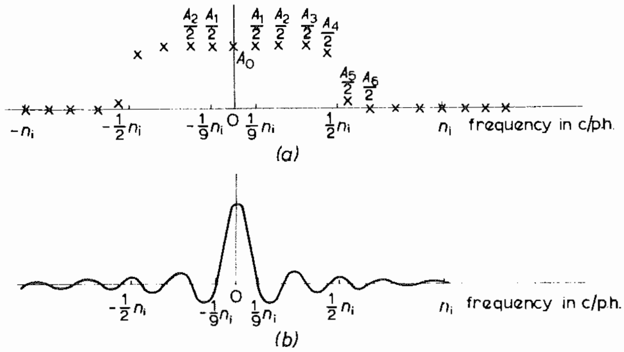

Fig. 15 - Derivation of spectrum of interpolation aperture of width $9/n_i$ p.h.

The required aperture with a spectrum passing through the points shown in Fig. 15(*a*) is given by one cycle of this repetitive waveform. The approximate shape of this aperture is shown in Fig. 16. The parts of the spectrum of the aperture between the points marked in Fig. 15(*a*) are given by replacing each point with a curve of the same shape as the spectrum of a square pulse of width $9/n_i$ p.h. (see Fig. 15(*b*)) and then summing all the curves.

It can be seen from the above that an aperture of width $9/n_i$ p.h. can have a spectrum which is much closer to the ideal spectrum than is possible with an aperture of width $2/n_i$ p.h.

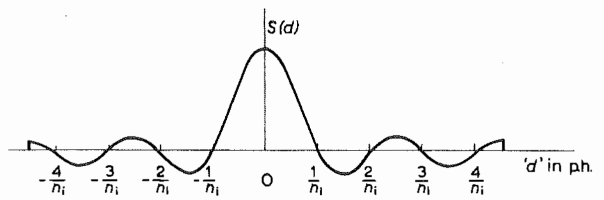

Fig. 16 - Shape required for interpolation aperture of width $9/n_i$ p.h.

As a general rule, the greater the width of an aperture the more closely we can approach the ideal spectrum but the more complex the instrumentation. The increased complexity of the instrumentation is largely due to the need to store a greater amount of information.

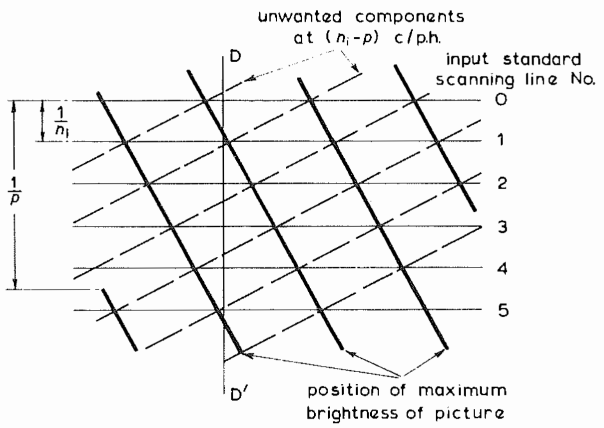

Fig. 17 - Positions of maximum brightness of picture

7. EFFECT OF UNWANTED COMPONENTS ON A DISPLAY OF A CONVERTED SIGNAL

When a converted signal is displayed as an image, the unwanted information introduced by the conversion process is most noticeable along sloping edges of objects in the scene. In order to analyse the spurious signals which result from sloping edges, it is convenient to consider an original scene consisting of diagonal stripes which vary sinusoidally in brightness across their width. Such a pattern is indicated by the full diagonal lines in Fig. 17. It will be assumed that this pattern is scanned at the input standard along the horizontal lines indicated. The signal corresponding to points at which scanning lines cross a narrow vertical strip of picture such as DD′ will now be considered. If the vertical spatial frequency of the diagonal pattern is p c/p.h., then the interpolated envelope of the line-samples obtained along DD′ has a spectrum similar to that shown in Fig. 18. In general,

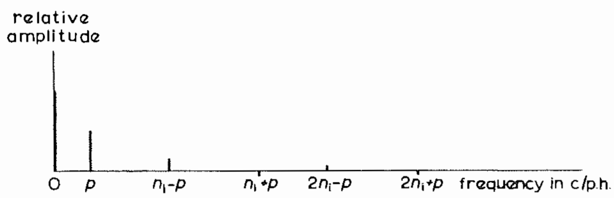

Fig. 18 - Spectrum of interpolated envelope of input line samples obtained along DD′ in Fig. 17

the unwanted component in this spectrum which has a frequency of $(n_i - p)$ c/p.h. has a greater amplitude than the higher frequency unwanted components and only this lowest frequency unwanted component will be considered further. By determining the relative phases of the wanted component at p c/p.h. and unwanted component at $(n_i - p)$ c/p.h. for varying positions of the line DD′, it can be shown that maxima of this unwanted component occur along lines indicated by the broken diagonal lines in Fig. 17. (These broken diagonal lines cross scanning lines at the same point as the wanted diagonal lines.)

It was shown in Section 3 that the converted signal is obtained by sampling the interpolated envelope of input line samples at the output standard. It therefore follows that a display of the converted signal obtained from the diagonal pattern being considered will appear to have been obtained by directly scanning a picture in which the original pattern has superimposed on it the pattern indicated by the broken diagonal lines in Fig. 17.

So far, only one field of the input standard has been considered. If the input standard is an interlaced system then the phase of the unwanted component at $(n_i - p)$ c/p.h. will change by 180° during alternate fields. As a result, the positions of maxima of this unwanted component during odd and even field periods are as shown in Fig. 19. Output scanning lines are also plotted on Fig. 19 for an interlaced output standard. It can be seen from this figure that the maximum amplitude of the unwanted pattern on a display of the converted signal will occur at the points marked by O's during even fields and by X's during odd fields of the two standards. The final subjective effect of the unwanted component at $(n_i - p)$ c/p.h. on a display of the converted signal when both input and output standards are interlaced systems is that this component causes a diagonal pattern indicated by the dotted diagonal lines on Fig. 19. This pattern does not change in phase during alternate fields and it has a vertical spatial frequency of $\left|\,|n_i - n_o| - p\,\right|$ c/p.h. This stationary pattern can be quite visible when the original scene contains information similar to the corner diagonals of test cards C or D.

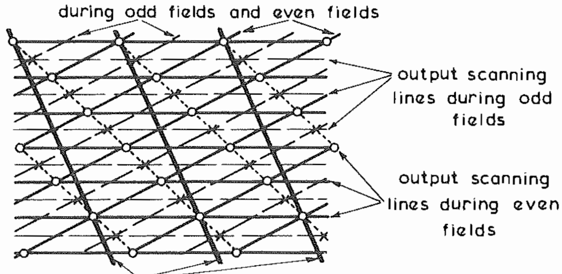

Fig. 19 - Pattern formed on display of converted signal by unwanted component shown in Fig. 18 when input and output standards are interlaced systems

o Points at which scanning lines cross unwanted pattern during even fields
x Points at which scanning lines cross unwanted pattern during odd fields

The patterns described above are quite visible in practice when a cosine-squared interpolating aperture is used, since with this aperture the amplitude of the unwanted component at $(n_i - p)$ c/p.h. is significantly greater than that of higher frequency unwanted components.

8. EXPERIMENT TO DETERMINE AN OPTIMUM INTERPOLATION APERTURE USING DIAGONAL TEST PATTERNS

8.1. Introduction

The purpose of the experiment described below was to determine the ratio of cosine to lift in a "cosine plus lift" interpolating aperture with a width of $2/n_i$ p.h.s. which was required for minimum visibility of spurious patterning on a display of a converted signal. The theory given earlier in this report indicates that the optimum ratio of cosine to lift should depend on the vertical frequency but not on the horizontal frequency of picture information. Experiments were, therefore, carried out with sinusoidal signals giving stationary patterns with varying horizontal and vertical spatial frequencies.

8.2. Apparatus

A block diagram of the equipment used in the experiments is shown in Fig. 20. All the experi-

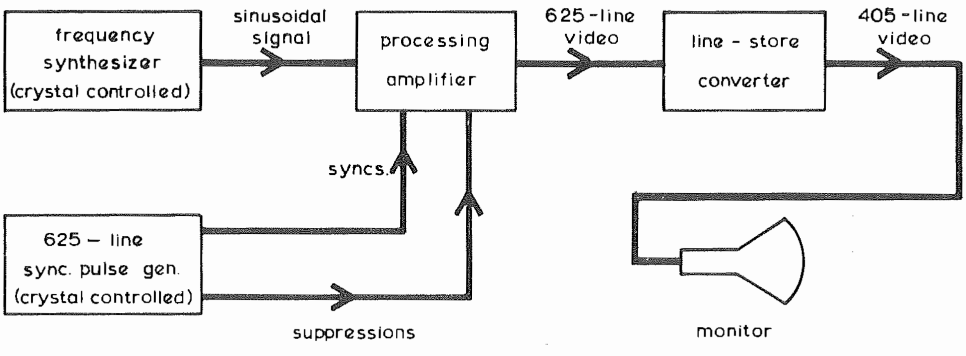

Fig. 20 - Block diagram of equipment used to determine optimum ratio of cosine/lift in interpolation aperture

ments were carried out on conversion from a 625-line video signal to a 405-line video signal. The method of interpolation used in the converter was based on the principles mentioned in Section 3.

As indicated in the block diagram, the converter was supplied with a sinusoidal video signal obtained by passing a sinusoidal signal through a processing amplifier which added blanking and synchronizing pulses. The resulting waveform during several line periods of the input signal was similar to that shown in Fig. 21.

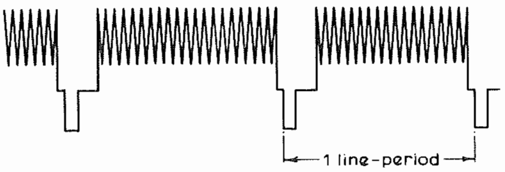

Fig. 21 - Waveform of a signal supplied to converter

The sine-wave generator used in the tests was a Rhode and Schwarz frequency synthesiser type XUA.

The frequencies of both the frequency synthesiser and the 625-line sync pulse generator were controlled by crystal oscillators in order that stable patterns could be produced on the display of the converted signal.

A "cosine plus lift" interpolating aperture, as shown in Fig. 11, was used in all the tests. The amplitude A_1 of the cosine component of the aperture could be varied independently of the amplitude A_0 of the lift component by adjusting a potentiometer in the converter; the resulting ratio A_1/A_0 could be read directly from a meter connected to the slider of this potentiometer.

8.3. Method

The frequency synthesiser was first adjusted to give a signal at a frequency of 'af_L' Hz, where 'a' was an integer of the order of 100, and 'f_L' was the line frequency of the synchronizing pulse generator (i.e. 15, 625 Hz).

Ignoring the picture lost during blanking periods, the display of the resulting video signal consisted of 'a' vertical bars as indicated in Fig. 22(a). Since these bars are vertical, alteration of the interpolation law has no effect on a display of the converted signal obtained in this way.

The frequency of the signal from the synthesiser was then altered by 'bf_F' Hz to $af_L \pm bf_F$ Hz, where b was an integer between 20 and 200, and f_F

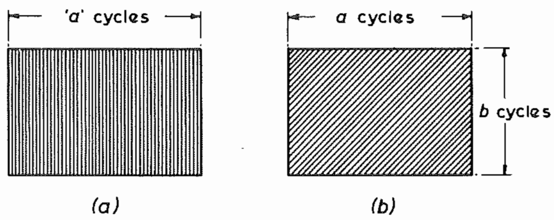

Fig. 22 - Wanted patterns produced on display at given frequencies of video signal
(a) frequency = af_L c/p.h.
(b) frequency = $af_L + bf_F$ c/p.h.

was the field frequency of the synchronizing pulse generator, i.e. 50 Hz.

Again ignoring the effects of blanking, the resulting display consisted of diagonal bars as indicated in Fig. 22(b), there being 'a' cycles per picture width and 'b' cycles per picture height: the direction in which the bars sloped depended on whether the frequency had been increased or decreased. It can be seen that this display is similar to that considered in Section 7.

Having obtained a display of the converted signal of the type illustrated in Fig. 22(b), the amplitude of the cosine component of the interpolation aperture was adjusted to give minimum visibility of unwanted information on the display of the converted signal and a note was made of the corresponding value of A_1/A_0.

The optimum value of A_1/A_0 was then found for varying values of 'a' and 'b'.

Before the interpolation aperture had been adjusted to give minimum spurious information of the display of the converted signal, the most noticeable feature of this spurious information was that it tended to produce a pattern of the type illustrated by the dotted lines in Fig. 19. As explained in Section 7, a pattern of this type was expected if the component of frequency (312½ - b) c/p.h. in the spectrum of the vertical information in the input signal was not removed by the interpolation process. In practice optimum interpolation appeared to correspond to minimum visibility of this particular pattern.

8.4. Results

A summary of results obtained is given in Table 1.

The values of A_1/A_0 given in Table 1 are the mean value of eight readings, four measurements being taken for a given positive value of 'b' and four for an equal negative value of 'b'.

TABLE 1

a (cycles per picture width)	b (cycles per picture height)	Frequency of output from synthesiser ($af_L + bf_F$) in kHz	Optimum ratio of cosine/lift A_1/A_0
80	± 40	1,250 ± 2	Approx. 0·7 (see Note 1)
80	± 60	1,250 ± 3	0·68
80	± 80	1,250 ± 4	0·50
80	± 100	1,250 ± 5	<0·5 (see Note 2)
120	± 40	1,875 ± 2	Approx. 0·7 (see Note 1)
120	± 60	1,875 ± 3	0·62
120	± 80	1,875 ± 4	0·60
120	± 100	1,875 ± 5	0·51
120	± 120	1,875 ± 6	0·41
120	± 140	1,875 ± 7	0·23
120	± 160	1,875 ± 8	<0·2 (see Note 2)
160	± 40	2,500 ± 2	Approx. 0·7 (see Note 1)
160	± 60	2,500 ± 3	0·64
160	± 80	2,500 ± 4	0·59
160	± 100	2,500 ± 5	0·56
160	± 120	2,500 ± 6	0·43
160	± 140	2,500 ± 7	0·22
160	± 160	2,500 ± 8	<0·2 (see Note 2)

Note 1: The adjustment of the interpolating aperture was not at all critical in these tests and was even less critical for lower values of 'b', i.e. for less vertical patterns. The value of A_1/A_0 obtained for b = 60 was quite suitable for all lower values of b.

Note 2: The wanted information was very difficult to distinguish from spurious information in these tests, and no interpolating aperture gave very satisfactory results.

The results given in Table 1 have been plotted in Fig. 23. In this Figure, the value of A_1/A_0 giving optimum interpolation has been plotted against 'b', rather than against 'a', since it was expected that A_1/A_0 would vary with the value of 'b' but not with the value of 'a'. Also plotted in Fig. 23 is a theoretical curve showing the required value of A_1/A_0 to cause the spectrum of the interpolation aperture to have a zero at (312½ - b) c/p.h.

8.5. Discussion of Results

The results plotted in Fig. 23 show that the value of A_1/A_0 required to give optimum interpolation depended mainly on the vertical spatial frequency of the picture information, and was only slightly affected by variations in the horizontal frequency. The variations in the required value of A_1/A_0 when the horizontal frequency of the patterns were altered might quite easily have been due to experimental errors.

Comparison of the experimental results with the theoretical curve in Fig. 23 showed that the value of A_1/A_0 for optimum interpolation was very closely equal to the value of A_1/A_0 required for the interpolation process to remove the lowest frequency unwanted component in the spectrum of the input line samples, i.e. the component at (312½ - b) c/p.h.

The experiment indicated that the best value of A_1/A_0 to keep the visibility of spurious patterns as small as possible over as wide a range of vertical frequencies as possible was of the order of 0·6.

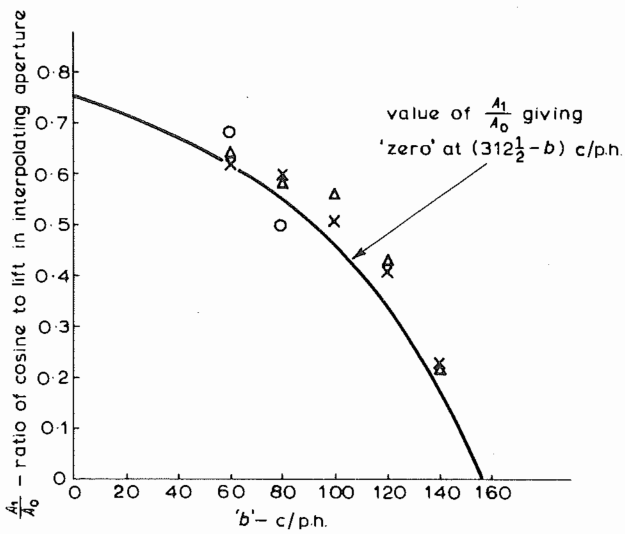

Fig. 23 - Ratio of cosine to lift for optimum interpolation

Optimum ratio of cosine to lift components in interpolation aperture is plotted against vertical spatial frequency 'b' of test patterns for various values of 'a', the horizontal spatial frequency

o a = 80 x a = 120 △ a = 160

9. OPTIMUM INTERPOLATION APERTURE WHEN CONVERTING NORMAL PICTURES

The effect on normal pictures and on test cards of using several different types of interpolation aperture of width equal to $2/n_i$ p.h. was investigated both for "up" and "down" conversions between 405-line and 625-line signals.

The interpolation apertures used in the tests may be divided into the following two types:

(*a*) cosine plus lift (see Fig. 11);

(*b*) triangular (see Fig. 9(*c*));

It was found, for both "up" and "down" conversion, that a "cosine plus lift" aperture having an optimum ratio of A_1/A_0 resulted in more acceptable converted pictures than those obtained when a triangular aperture was used. There appeared to be little difference, however, between results obtained with the optimum "cosine plus lift" aperture, and an aperture obtained by the adding of a small amount of lift component to the triangular aperture.

During "down" conversion, from a 625-line to a 405-line standard the optimum ratio of A_1/A_0 was found to be about 0·6 as in the experiments described in Section 8. This value of A_1/A_0 also gave near optimum interpolation for "up" conversion from a 405-line to a 625-line standard, although a value of 0·7 appeared to give slightly more satisfactory results.

Photographs illustrating the effect of interpolation are shown in Figs. 24 and 25. Fig. 24 shows a 405-line display of a signal converted from a 625-line signal without interpolation. In this method of standards conversion, the magnitude of the output signal at point Q in Fig. 1 would be equal to the magnitude of the input signal at point P_n. As a result, the information from 220 of the input lines has been lost in the conversion process. Fig. 25 shows a display obtained in a similar manner to that shown in Fig. 24 with the exception that the "cosine plus lift" interpolation aperture with $A_1/A_0 = 0·6$ was used in the standards conversion process.

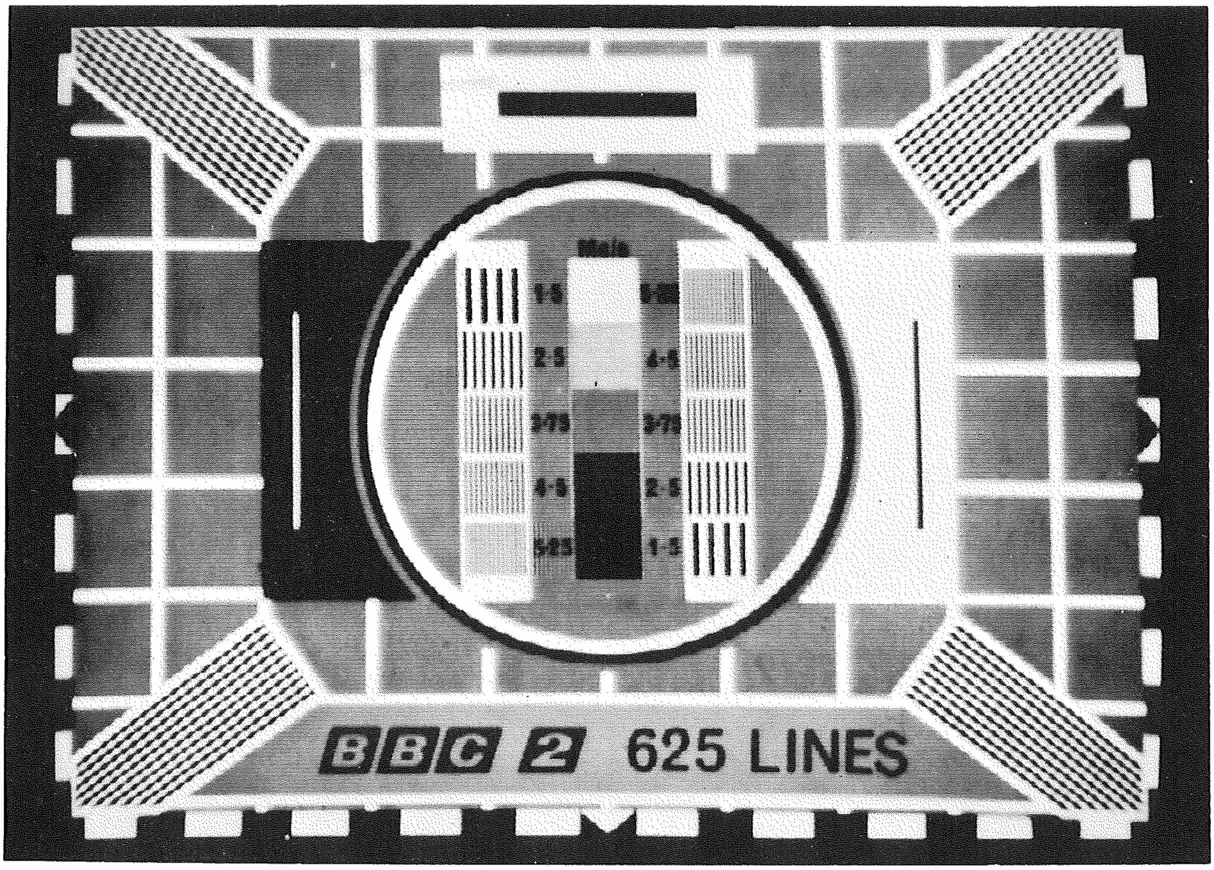

Fig. 24 - Test Card converted from 625 to 405 lines without interpolation

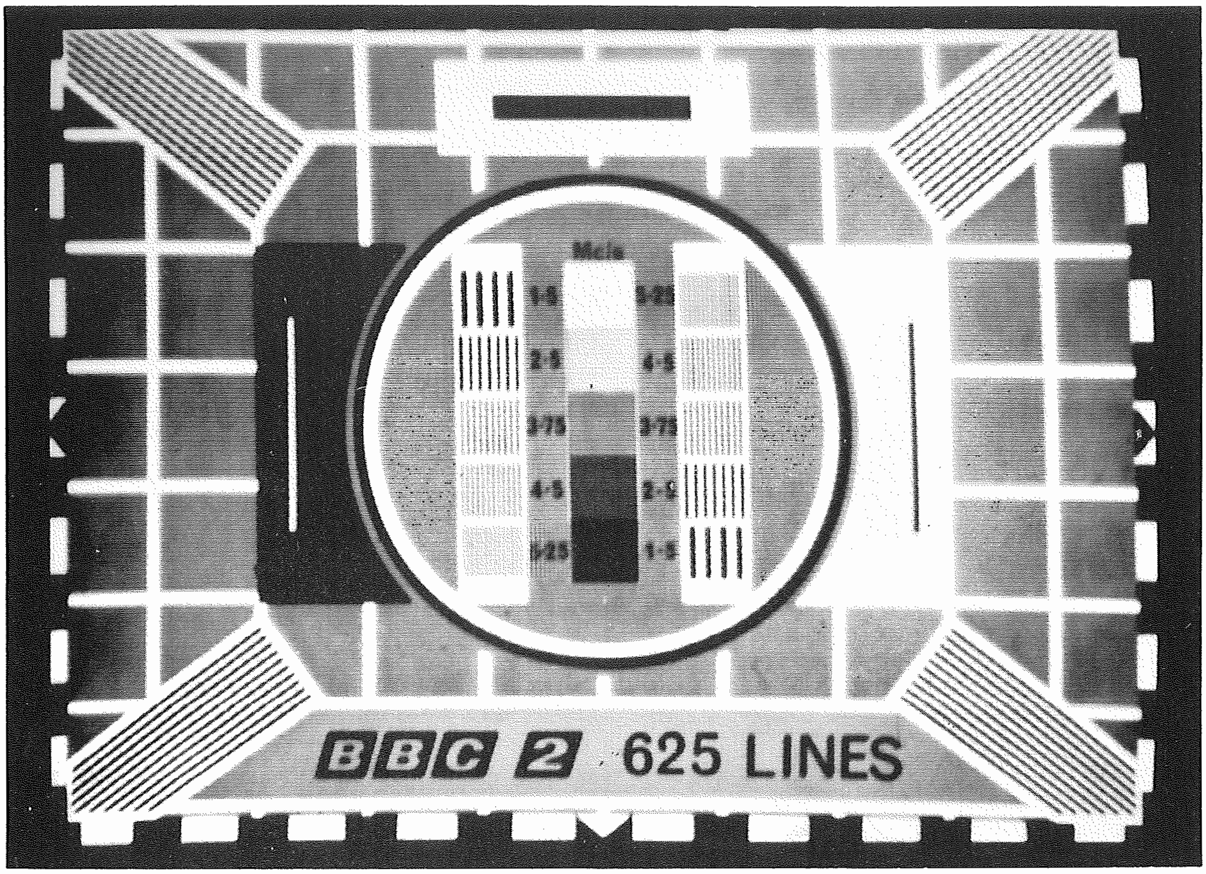

Fig. 25 - Test Card converted from 625 to 405 lines using "cosine plus lift" interpolation aperture with $A_1/A_0 = 0{\cdot}6$

10. CONCLUSIONS

From both theoretical and practical considerations it would appear that, if the interpolation in a line-store standards conversion is carried out between only two input lines at a time, then for minimum spurious patterning on a display of a converter signal a "cosine plus lift" interpolation aperture (see Fig. 11) for which A_1/A_0 equals approximately 0·6 gives results which are unlikely to be significantly improved upon by any other aperture which is limited in width in the same way. The exact value of A_1/A_0 required for optimum interpolation with this type of aperture appears, in practice, to depend slightly on the number of lines in the input and output standards of the converter, 0·6 being about the optimum value for 625-line to 405-line conversion, and 0·7 being about the optimum value for 405-line to 625-line conversion.

Better interpolation can be obtained if the information on more than two input lines is available for deriving the information at any point on an output line, but the instrumentation involved becomes correspondingly more elaborate at the number of input lines which are used increases. If a very large number of input lines were available, the optimum shape of the interpolation aperture would be similar to that shown in Fig. 8.

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